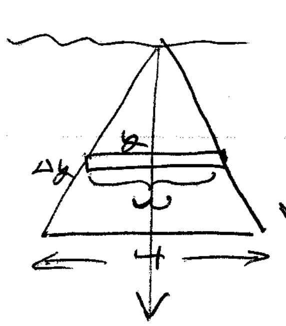
6. From the picture, $F = \int_0^5 \rho g y w \, dy$

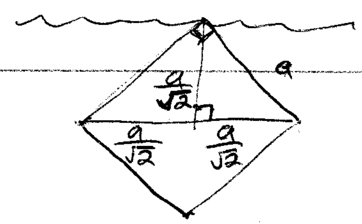


where $w = \frac{4}{5}y$, so

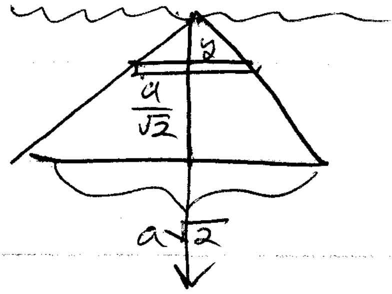
$$F = \int_0^5 (1000)(9.8)y\left(\frac{4}{5}y\right) dy = \frac{7840}{3} y^3 \Big|_0^5$$

$$= \boxed{\frac{980{,}000}{3} \text{ N}}$$

10. The picture is a bit vague, but I'm assuming this is a square. I'm not assuming any particular system of units, however. I broke the problem into two simpler problems, the top & the bottom of the square. Note: I set up DIFFERENT coordinate systems for each half.



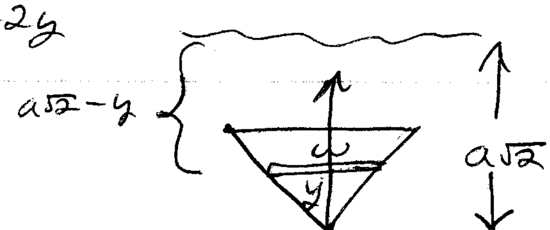
$$F_{Top} = \int_0^{a/\sqrt{2}} \rho g y w \, dy, \text{ where } w = 2y$$



$$= 2\rho g \int_0^{a/\sqrt{2}} y^2 \, dy = \frac{2}{3}\rho g y^3 \Big|_0^{a/\sqrt{2}}$$

$$= \frac{1}{3\sqrt{2}} \rho g a^3$$

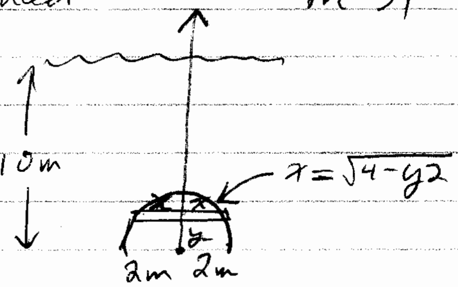
$$F_{Bottom} = \int_0^{a/\sqrt{2}} \rho g (a\sqrt{2} - y) w \, dy, \quad w = 2y$$



$$= 2\rho g \int_0^{a/\sqrt{2}} (a\sqrt{2}\, y - y^2) \, dy$$

$$= 2\rho g \left[\frac{a}{\sqrt{2}} y^2 - \frac{1}{3}y^3\right]_0^{a/\sqrt{2}} = \frac{4}{3}\rho g \left(\frac{a^3}{2\sqrt{2}}\right) = \frac{2\rho g a^3}{3\sqrt{2}}$$

Finally, $F = F_{Top} + F_{Bottom} = \frac{\rho g a^3}{3\sqrt{2}}(1+2) = \boxed{\frac{\rho g a^3}{\sqrt{2}}}$



14. $F = \int_0^2 (1000)(9.8)(10-y)2\sqrt{4-y^2}\,dy$

$= 196000\int_0^2 \sqrt{4-y^2}\,dy - 9800\int_0^2 (2y\sqrt{4-y^2})\,dy$

$= 196000\left(\frac{1}{4}\pi 2^2\right) - 9800\int_0^2 2y\sqrt{4-y^2}\,dy$

Let $u = 4-y^2$
$du = -2y\,dy$
$-du = 2y\,dy$

$= 196000\pi - 9800\int_0^4 \sqrt{u}\,du$

$= 196000\pi - 9800\cdot\frac{2}{3}u^{3/2}\Big|_0^4$

$= \boxed{196000\pi - \frac{156800}{3}} \approx 563485.5\ N$

22. $\bar{x} = \frac{-3m_1 + 2m_2 + 8m_3}{m_1+m_2+m_3} = \frac{-3\cdot 12 + 2\cdot 15 + 8\cdot 20}{12+15+20}$

$= \boxed{\frac{154}{47}}$

26. $A = \int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3}$

$\int_0^4 x f(x)\,dx = \int_0^4 x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^4 = \frac{64}{5}$

$\therefore\ \bar{x} = \frac{64/5}{16/3} = \frac{12}{5} = 2.4$

$\int_0^4 \frac{1}{2}[f(x)]^2\,dx = \frac{1}{2}\int_0^4 x\,dx = \frac{1}{4}x^2\Big|_0^4 = 4$

$\therefore\ \bar{y} = \frac{4}{16/3} = \frac{3}{4} = 0.75$ centroid: $\boxed{\left(\frac{12}{5}, \frac{3}{4}\right)}$

30. $A = \int_{-2}^1 [(2-x^2)-x]\,dx = \int_{-2}^1 (2-x-x^2)\,dx = \left[2x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{-2}^1 = \frac{9}{2}$

$\int_{-2}^1 x[(2-x^2)-x]\,dx = \int_{-2}^1 (2x-x^2-x^3)\,dx = -\frac{9}{4}$

so $\bar{x} = \frac{-9/4}{9/2} = \boxed{-\frac{1}{2}}$

$\int_{-2}^1 \frac{1}{2}\left[(2-x^2)^2 - x^2\right]dx = \frac{1}{2}\int_{-2}^1 [(4-4x^2+x^4)-x^2]\,dx = \frac{1}{2}\int_{-2}^1 (4-5x^2+x^4)\,dx$

$= \frac{1}{2}\left[4x - \frac{5}{3}x^3 + \frac{1}{5}x^5\right]_{-2}^1 = \frac{9}{5}$, $\therefore\ \bar{y} = \frac{9/5}{9/2} = \boxed{\frac{2}{5}}$ centroid: $\boxed{\left(-\frac{1}{2}, \frac{2}{5}\right)}$